Optimal stopping of Markov chains
or
How to play Blackjack

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Blackjack is a gambling game between you and the dealer. The basic idea is to get a higher total than the dealer, without exceeding 21.

I cannot tell you how to win every hand. What I can do is to tell you how to figure out a strategy to maximize the average amount you win on every bet (actually, minimize the amount you lose!).

The strategy that appears at the end of this talk is offered without warranty. You are at your own risk if you use it.

Card values

- Cards numbered 2 to 9 are worth 2 to 9.
- 10, Jack, Queen, and King are worth 10; they are called *face cards*.
- Ace is worth 11 (if this doesn't make your total exceed 21) or 1 (if counting it as 11 would make your total exceed 21). These are called *soft* and *hard* totals, respectively.
- A face card and an ace is called *Blackjack*. Blackjack beats a total of 21 obtained any other way.

How each hand is played

- The dealer gives himself one card, face up.
- The dealer gives you two cards, face up.
- Based on this information, you need to decide whether to be dealt another card (*hit*) or to stop taking cards (*stand*).
- If you hit and this makes your total exceed 21, you have gone *bust* and the hand ends.
- If you do not bust, you may hit again, as many times as you like, provided that you don't bust.
- (Doubling down and splitting are more complicated options.)
- Once you stand, the dealer deals himself cards according to the **Dealer's Strategy**: hit if the total is 16 or less, stand on a total of 17 or more, even a soft total.

The payoff

Assume that each hand of cards is worth \$1.

- If you go bust or the dealer gets a larger total than you, you give the dealer \$1.
- If you get blackjack and the dealer doesn't, you get \$1.50.
- If the dealer goes bust or gets a smaller total than yours, you get \$1.
- If you and the dealer tie, no money is exchanged.

Rules vary to some extent between different casinos.

Dealer's cards

Your cards

A deck of cards

Casinos used to use just one deck of cards.

By keeping track of what cards have already been dealt (*counting cards*), you could change your strategy and improve your chances of winning.

For example, if your total is 16, you may be reluctant to hit. However, if very few small cards have been dealt, you may choose to hit, reasoning that the next card is more likely to be small.

In the 1960's, a mathematician named Edward O. Thorp used computer simulations to devise a good way to count and a good hitting strategy. He used these and other people's money to win a lot of money, for a while. You can read about it in his book *Beat the Dealer* (Random House, New York, 1966).

Then the casinos changed the rules to make counting more difficult. Now, at each Blackjack table, the dealer has 6 or 8 decks of cards shuffled together, and once about 2/3 of the cards are used, the dealer gets a brand new set of shuffled cards.

This makes life much harder for card counters, but it makes the following assumption more reasonable.

Independence assumption

The cards that are dealt are independent and identically distributed:

- For each card that is dealt,

  The probability it is an Ace is $\frac{1}{13}$

  The probability it is a 2 is $\frac{1}{13}$ ...

  The probability it is a 9 is $\frac{1}{13}$

  The probability it is a Face card is $\frac{4}{13}$

- These probabilities are not affected by what has been dealt already. For instance, if 20 Aces have been dealt in a row, the next card will still be an Ace with probability $\frac{1}{13}$.

Markov chain

Suppose I know what total the player has, for example, hard 17, soft 14, Blackjack. It doesn't matter what the individual cards are.

When the player hits, I can compute the probabilities that his total will become, say, hard 20, soft 21, hard 21, because the next card to be drawn is independent of the cards that were drawn before.

The player's total is a **Markov chain**. The same is true of the dealer's total. The only difference between the two is that the dealer's strategy is fixed by the rules of the game, while the player may choose whatever strategy he likes.

The dealer's total

Suppose the dealer begins with a 5. By following the dealer's strategy, he will end up with 17, 18, 19, 20, 21, Blackjack, or Bust, with certain probabilities. (The probability of Blackjack is zero.)

We need to compute these probabilities before we can devise our strategy.

When the dealer draws a card, his total can become 7, 8, 9, 10, 11, 12, 13, 14, or soft 16, each with probability $\frac{1}{13}$, or it could be 15, with probability $\frac{4}{13}$. Then he will need to draw another card, then maybe another.

The possible totals the dealer can ever have are:

```
    <---------------- Hard totals ------------------------->     <-------- Soft totals --------->
 0  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 Face 11 12 13 14 15 16 17 18 19 20 21  BJ Bust
```

Starting with a 5, the probabilities of getting one of these totals after drawing one card are, in thirteenths:

```
    <---------------- Hard totals ------------------------->     <-------- Soft totals --------->
 0  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 Face 11 12 13 14 15 16 17 18 19 20 21  BJ Bust
 0  0  0  0  0  0  1  1  1  1  1  1  1  1  4  0  0  0  0  0  0 : 0 :  0  0  0  0  0  1  0  0  0  0  0 : 0 :  0
```

The Dealer's transition matrix D

Whatever total the dealer has, given another opportunity to draw a card, the probabilities of gettings the various possible new totals may be organized into a 35 by 35 matrix, in thirteenths. Blank entries are zeros.

	<----------------- Hard totals ------------------------->																						<-------- Soft totals --------->												
	0	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	Face	11	12	13	14	15	16	17	18	19	20	21	BJ	Bust
0		1	1	1	1	1	1	1	1													4	1												
2				1	1	1	1	1	1	1	1	4													1										
3					1	1	1	1	1	1	1	1	4													1									
4						1	1	1	1	1	1	1	1	4													1								
5							1	1	1	1	1	1	1	1	4													1							
6								1	1	1	1	1	1	1	1	4													1						
7									1	1	1	1	1	1	1	1	4													1					
8										1	1	1	1	1	1	1	1	4													1				
9											1	1	1	1	1	1	1	1	4													1			
10												1	1	1	1	1	1	1	1	4													1		
11												1	1	1	1	1	1	1	1	1	4														
12													1	1	1	1	1	1	1	1	1														4
13														1	1	1	1	1	1	1	1														5
14															1	1	1	1	1	1	1														6
15																1	1	1	1	1	1														7
16																	1	1	1	1	1														8
17																	13																		
18																		13																	
19																			13																
20																				13															
21																					13														
Face												1	1	1	1	1	1	1	1	4														1	
S 11																								1	1	1	1	1	1	1	1	1		4	
S 12												4													1	1	1	1	1	1	1	1	1		
S 13												1	4													1	1	1	1	1	1	1	1		
S 14												1	1	4													1	1	1	1	1	1	1		
S 15												1	1	1	4													1	1	1	1	1	1		
S 16												1	1	1	1	4													1	1	1	1	1		
S 17																													13						
S 18																														13					
S 19																															13				
S 20																																13			
S 21																																	13		
BJ																																		13	
Bust																																			13

Note: beginning with a total of 17, 18, 19, 20, 21, Blackjack, or Bust, the dealer's total will not change.

Before the dealer deals himself a card, his total is zero.

Transition probabilities after drawing two cards

Suppose the dealer begins with a 5. He will draw one card and then need to draw another. Using the matrix D, we can compute the probabilities of the dealer ending up with various totals after drawing two cards, as follows.

We look at all possible intermediate totals `x`, multiply the probability of going from 5 to `x` times the probability of going from `x` to 17, then add these up.

```
      x x x x x x x x x x x x x x x x x x                  17
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
  5 | * * * * * * * * * * * * * * * * * * |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
    |                                     |    x |          *          |
```

Thus, starting with a 5 and having two opportunities to draw a card, the probability of ending up with a total of 17 is found in row 5, column 17 of the matrix D^2.

The matrix D^2 represents transition probabilities after two opportunities to draw a card. Similarly, D^3 gives transition probabilities after three opportunities, etc.

Final transition probabilities for the dealer

After 20 chances to draw, the dealer **must** have either gone bust or arrived at a total at which he must stand. The matrix D^{20} contains all the relevant transition probabilities. The relevant entries are below:

		Bust	17	18	19	20	21	Blackjack
	2	0.3536	0.1398	0.1349	0.1297	0.1240	0.1180	0
	3	0.3739	0.1350	0.1305	0.1256	0.1203	0.1147	0
	4	0.3945	0.1305	0.1259	0.1214	0.1165	0.1112	0
Dealer's	5	0.4164	0.1223	0.1223	0.1177	0.1131	0.1082	0
up	6	0.4232	0.1654	0.1063	0.1063	0.1017	0.0972	0
card	7	0.2623	0.3686	0.1378	0.0786	0.0786	0.0741	0
	8	0.2447	0.1286	0.3593	0.1286	0.0694	0.0694	0
	9	0.2284	0.1200	0.1200	0.3508	0.1200	0.0608	0
	Face	0.2121	0.1114	0.1114	0.1114	0.3422	0.0345	0.0769
	Ace	0.1153	0.1308	0.1308	0.1308	0.1308	0.0539	0.3077

Expected winnings if you stand

Suppose the dealer has a 5 and you stand on a total of 11. How much do you expect to win (or lose)?

You win only if the dealer busts, so on average, you win:

$$\begin{aligned} & 1 \cdot \mathbb{P}(\text{Dealer busts}) + -1 \cdot \mathbb{P}(\text{Dealer doesn't bust}) \\ &= 1 \cdot 0.4164 + -1 \cdot 0.5836 \\ &= -0.1672. \end{aligned}$$

Thus, on average, you would lose 16.72 cents per hand in this situation if you followed this strategy.

Expected winnings matrix F

For each total you might have and each card the dealer may have, you can compute the expected winnings if you stand.

	Dealer's up card									
	2	3	4	5	6	7	8	9	Face	Ace
2	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
3	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
4	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
5	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
6	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
7	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
8	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
9	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
10	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
11	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
12	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
13	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
14	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
15	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
16	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
17	-0.1530	-0.1172	-0.0806	-0.0449	0.0117	-0.1068	-0.3820	-0.4232	-0.4644	-0.6386
18	0.1217	0.1483	0.1759	0.1996	0.2834	0.3996	0.1060	-0.1832	-0.2415	-0.3771
19	0.3863	0.4044	0.4232	0.4395	0.4960	0.6160	0.5939	0.2876	-0.0187	-0.1155
20	0.6400	0.6503	0.6610	0.6704	0.7040	0.7732	0.7918	0.7584	0.4350	0.1461
21	0.8820	0.8853	0.8888	0.8918	0.9028	0.9259	0.9306	0.9392	0.8117	0.3307
Face	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.4989	-0.4617
S 11	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
S 12	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
S 13	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
S 14	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
S 15	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
S 16	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4754	-0.5105	-0.5431	-0.5758	-0.7694
S 17	-0.1530	-0.1172	-0.0806	-0.0449	0.0117	-0.1068	-0.3820	-0.4232	-0.4644	-0.6386
S 18	0.1217	0.1483	0.1759	0.1996	0.2834	0.3996	0.1060	-0.1832	-0.2415	-0.3771
S 19	0.3863	0.4044	0.4232	0.4395	0.4960	0.6160	0.5939	0.2876	-0.0187	-0.1155
S 20	0.6400	0.6503	0.6610	0.6704	0.7040	0.7732	0.7918	0.7584	0.4350	0.1461
S 21	0.8820	0.8853	0.8888	0.8918	0.9028	0.9259	0.9306	0.9392	0.8117	0.3307
BJ	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.3846	1.0385
Bust	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000

The player's total

Now that we know the consequences of standing, we can consider the consequences of hitting.

The player's total is also a Markov chain. The transition matrix P is slightly different from the dealer's in that it doesn't force the player to stand on 17, 18, 19, $\ldots$.

Every time the player hits, his total takes one step according to the transition matrix P.

The question is when the player should stop.

Optimal stopping

Suppose the dealer has a 5 and the player's total is a hard 11. He should hit because he is guaranteed a higher total without busting.

Suppose the dealer has a 5 and the player's total is a hard 12. If he hits, there is the possibility of busting. But if he doesn't bust, he will have a higher total and be more likely to beat the dealer.

We can compute the expected winnings from hitting once by computing another matrix product, this time P times F. The entries of this matrix are similar to those of F, but represent the expected winnings after hitting once and then standing.

If an entry of PF is larger than the corresponding entry of F, then it is advantageous to hit in that situation. Where PF is smaller than F, it is advantageous to stop.

The value matrix V

Consider the matrix `V = max(F,PF)`, where `max` means to take the maximum of the two matrices entry by entry. Then each entry of `V` represents the expected winnings if you make the right choice between standing and hitting one more time.

But it might be necessary to hit more than one time, so we can repeat this procedure, at each stage replacing `V` by `max(V,PV)` to represent making the correct choice between standing or hitting at least one more time.

Repeating this 10 times, the matrix `V` stops changing and becomes:

	Dealer's up card									
	2	3	4	5	6	7	8	9	Face	Ace
2	-0.0759	-0.0498	-0.0221	0.0137	0.0389	-0.0273	-0.1032	-0.1900	-0.3003	-0.4485
3	-0.1005	-0.0689	-0.0363	0.0002	0.0245	-0.0574	-0.1309	-0.2151	-0.3218	-0.4655
4	-0.1149	-0.0826	-0.0494	-0.0124	0.0111	-0.0883	-0.1593	-0.2407	-0.3439	-0.4829
5	-0.1282	-0.0953	-0.0615	-0.0240	-0.0012	-0.1194	-0.1881	-0.2666	-0.3662	-0.5006
6	-0.1408	-0.1073	-0.0729	-0.0349	-0.0130	-0.1519	-0.2172	-0.2926	-0.3887	-0.5183
7	-0.1092	-0.0766	-0.0430	-0.0073	0.0292	-0.0688	-0.2106	-0.2854	-0.3714	-0.5224
8	-0.0218	0.0080	0.0388	0.0708	0.1150	0.0822	-0.0599	-0.2102	-0.3071	-0.4441
9	0.0744	0.1013	0.1290	0.1580	0.1960	0.1719	0.0984	-0.0522	-0.2181	-0.3532
10	0.1825	0.2061	0.2305	0.2563	0.2878	0.2569	0.1980	0.1165	-0.0536	-0.2513
11	0.2384	0.2603	0.2830	0.3073	0.3337	0.2921	0.2300	0.1583	0.0334	-0.2087
12	-0.2534	-0.2337	-0.2111	-0.1672	-0.1537	-0.2128	-0.2716	-0.3400	-0.4287	-0.5504
13	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.2691	-0.3236	-0.3872	-0.4695	-0.5825
14	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.3213	-0.3719	-0.4309	-0.5074	-0.6123
15	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.3698	-0.4168	-0.4716	-0.5425	-0.6400
16	-0.2928	-0.2523	-0.2111	-0.1672	-0.1537	-0.4148	-0.4584	-0.5093	-0.5752	-0.6657
17	-0.1530	-0.1172	-0.0806	-0.0449	0.0117	-0.1068	-0.3820	-0.4232	-0.4644	-0.6386
18	0.1217	0.1483	0.1759	0.1996	0.2834	0.3996	0.1060	-0.1832	-0.2415	-0.3771
19	0.3863	0.4044	0.4232	0.4395	0.4960	0.6160	0.5939	0.2876	-0.0187	-0.1155
20	0.6400	0.6503	0.6610	0.6704	0.7040	0.7732	0.7918	0.7584	0.4350	0.1461
21	0.8820	0.8853	0.8888	0.8918	0.9028	0.9259	0.9306	0.9392	0.8117	0.3307
Face	0.2300	0.2534	0.2775	0.3030	0.3337	0.3011	0.2418	0.1597	-0.0095	-0.1969
S 11	0.5598	0.5768	0.5944	0.6129	0.6396	0.6340	0.5759	0.4940	0.3431	0.1168
S 12	0.0818	0.1035	0.1266	0.1565	0.1860	0.1655	0.0951	0.0001	-0.1415	-0.3219
S 13	0.0466	0.0741	0.1025	0.1334	0.1617	0.1224	0.0541	-0.0377	-0.1737	-0.3474
S 14	0.0224	0.0508	0.0801	0.1119	0.1392	0.0795	0.0133	-0.0752	-0.2057	-0.3727
S 15	-0.0001	0.0292	0.0593	0.0920	0.1182	0.0370	-0.0271	-0.1122	-0.2373	-0.3977
S 16	-0.0210	0.0091	0.0400	0.0734	0.0988	-0.0049	-0.0668	-0.1486	-0.2684	-0.4224
S 17	-0.0005	0.0290	0.0593	0.0912	0.1281	0.0538	-0.0729	-0.1498	-0.2586	-0.4320
S 18	0.1217	0.1483	0.1759	0.1996	0.2834	0.3996	0.1060	-0.1007	-0.2097	-0.3720
S 19	0.3863	0.4044	0.4232	0.4395	0.4960	0.6160	0.5939	0.2876	-0.0187	-0.1155
S 20	0.6400	0.6503	0.6610	0.6704	0.7040	0.7732	0.7918	0.7584	0.4350	0.1461
S 21	0.8820	0.8853	0.8888	0.8918	0.9028	0.9259	0.9306	0.9392	0.8117	0.3307
BJ	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.3846	1.0385
Bust	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000

The optimal stopping strategy

Where the value `V` from continuing to hit exceeds the expected winnings `F` from standing, one should hit. These places are shown by the letter `H` below. For example, if the dealer has a 5 and you have a hard 12, you stand.

Player's total	Dealer's up card 2	3	4	5	6	7	8	9	Face	Ace
2	H	H	H	H	H	H	H	H	H	H
3	H	H	H	H	H	H	H	H	H	H
4	H	H	H	H	H	H	H	H	H	H
5	H	H	H	H	H	H	H	H	H	H
6	H	H	H	H	H	H	H	H	H	H
7	H	H	H	H	H	H	H	H	H	H
8	H	H	H	H	H	H	H	H	H	H
9	H	H	H	H	H	H	H	H	H	H
10	H	H	H	H	H	H	H	H	H	H
11	H	H	H	H	H	H	H	H	H	H
12	H	H				H	H	H	H	H
13						H	H	H	H	H
14						H	H	H	H	H
15						H	H	H	H	H
16						H	H	H	H	H
17										
18										
19										
20										
21										
Face	H	H	H	H	H	H	H	H	H	H
S 11	H	H	H	H	H	H	H	H	H	H
S 12	H	H	H	H	H	H	H	H	H	H
S 13	H	H	H	H	H	H	H	H	H	H
S 14	H	H	H	H	H	H	H	H	H	H
S 15	H	H	H	H	H	H	H	H	H	H
S 16	H	H	H	H	H	H	H	H	H	H
S 17	H	H	H	H	H	H	H	H	H	H
S 18								H	H	H
S 19										
S 20										
S 21										
BJ										
Bust										

Doubling down

You may choose to double your bet and draw exactly one more card, then stand. In certain cases this is to your advantage; these are shown by `D` below.

	Dealer's up card									
Player's total	2	3	4	5	6	7	8	9	Face	Ace
2	H	H	H	H	H	H	H	H	H	H
3	H	H	H	H	H	H	H	H	H	H
4	H	H	H	H	H	H	H	H	H	H
5	H	H	H	H	H	H	H	H	H	H
6	H	H	H	H	H	H	H	H	H	H
7	H	H	H	H	H	H	H	H	H	H
8	H	H	H	H	H	H	H	H	H	H
9	H	D	D	D	D	H	H	H	H	H
10	D	D	D	D	D	D	D	D	H	H
11	D	D	D	D	D	D	D	D	H	H
12	H	H				H	H	H	H	H
13						H	H	H	H	H
14						H	H	H	H	H
15						H	H	H	H	H
16						H	H	H	H	H
17										
18										
19										
20										
21										
Face	D	D	D	D	D	D	D	D	H	H
S 11	D	D	D	D	D	D	D	D	D	H
S 12	H	H	H	H	H	H	H	H	H	H
S 13	H	H	H	H	D	H	H	H	H	H
S 14	H	H	H	D	D	H	H	H	H	H
S 15	H	H	H	D	D	H	H	H	H	H
S 16	H	H	D	D	D	H	H	H	H	H
S 17	H	D	D	D	D	H	H	H	H	H
S 18		D	D	D	D			H	H	H
S 19										
S 20										
S 21										
BJ										
Bust										

Splitting

If you are dealt a pair, you may split it to form two separate hands, each with the same amount at stake as the original bet. The situations in which you should do this are shown by S below.

Player's pair	Dealer's up card 2	3	4	5	6	7	8	9	Face	Ace
2	S	S	S	S	S	S				
3	S	S	S	S	S	S				
4				S	S					
5										
6	S	S	S	S	S					
7	S	S	S	S	S	S				
8	S	S	S	S	S	S	S	S		
9	S	S	S	S	S		S	S		
Face										
Ace	S	S	S	S	S	S	S	S	S	

The value of a hand of Blackjack

The amount you will win or lose on average depends on the strategy you use.

```
Following dealer's strategry :  lose 5.67 cents per $1 bet
Optimal stopping alone       :  lose 2.42 cents per $1 bet
With doubling                :  lose 1.17 cents per $1 bet
With splitting               :  lose 0.68 cents per $1 bet
```

Remember that the rules of Blackjack are set to favor the casino. Even if you play optimally, you will lose, on average. If you have less money than the casino, you will tend to go bankrupt long before the casino does.

Your only hope is to:

A) Not play Blackjack for real money

B) Count cards. But beware! It is hard to do and it is easy to fool yourself into thinking you are doing it right. There are many bankrupt card counters!